

Beauty of Mathematics Decal PSET #9 Solutions

Due 11/15

Let's consider the integers mod 7, which consists of the numbers $\{0, 1, 2, 3, 4, 5, 6\}$.

1. Remember we should be able to divide; so which of these numbers is "1/2"? (That is, which number can we multiply by 2 to get 1?)

Let's compute all the multiples of 2.

$$0 * 2 = 0$$

$$1 * 2 = 2$$

$$2 * 2 = 4$$

$$3 * 2 = 6$$

$$4 * 2 = 1 \text{ (of course } 4*2=8 \text{ but } 8 \text{ reduces to } 1 \text{ mod } 7)$$

$$5 * 2 = 3 \text{ (etc.)}$$

$$6 * 2 = 5$$

Since $4 * 2 = 1$, we see that "1/2" is 4.

2. Some of these numbers are squares, and others are not. Show that 3 is a "square root" of 2, but that 5 has no square root.

Let's compute all the squares.

$$0^2 = 0$$

$$1^2 = 1$$

$$2^2 = 4$$

$$3^2 = 2 \text{ (of course } 3^2 = 9, \text{ but } 9 \text{ reduces to } 2 \text{ mod } 7)$$

$$4^2 = 2 \text{ (etc.)}$$

$$5^2 = 4$$

$$6^2 = 1$$

Observe that 5 is not the square of anything, so it has no square root; but $3^2 = 2$ so 3 is a "square root" of 2.

Now let's instead consider the integers mod 6, consisting of $\{0, 1, 2, 3, 4, 5\}$.

3. Show that there is no “ $1/2$ ” here; that is, multiplying something by 2 will never give 1.

Let’s compute all the multiples of 2.

$$0 * 2 = 0$$

$$1 * 2 = 2$$

$$2 * 2 = 4$$

$$3 * 2 = 0 \text{ (of course } 3 * 2 = 6 \text{ but } 6 \text{ reduces to } 0 \text{ mod } 6)$$

$$4 * 2 = 2 \text{ (etc.)}$$

$$5 * 2 = 4$$

We see that nothing multiplies 2 to make 1, so there is no “ $1/2$ ” here.

Observe that we can’t necessarily divide mod 6, but we can divide mod 7. This is because 7 is prime and 6 is not.